

Algebraic Method.

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3$$

$$\text{subject to } x_1 + 2x_2 + x_3 \leq 430 \quad \text{availability time on m/c 1}$$

$$3x_1 + 2x_3 \leq 460 \quad \text{" " " 2}$$

$$x_1 + 4x_2 \leq 420 \quad \text{" " " 3}$$

$$x_1, x_2, x_3 \geq 0$$

The co-efficient of slack variables represent the dual price.

In case of minimization, co-efficient of slack variables represent the dual price regardless of the sign.

In case of surplus, change the sign of co-efficient of surplus variable.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	solution
z	-3	-2	-5	0	0	0	0
x_4	1	2	1	1	0	0	430
x_5	3	0	2	0	1	0	460
x_6	1	4	0	0	0	1	420

$$(\text{New Value} - \text{Original Value}) = D_i$$

D value is actually the change.

Slack = 20 means $\frac{1}{2}$ of Resource 1 is

used $\frac{1}{2} \times 400 = 200$

$$\text{If } D_1 = 20, \quad D_2 = 0, \quad D_3 = 0$$

$$Z = 1370$$

$$x_2 = 110$$

$$x_3 = 230$$

$x_6 = -20$ \rightarrow shows that solution is infeasible.

We have to come up with two range

- 1) Individual Range
- 2) Simultaneous Range

$$20 = 2D_1$$

$$D_1 = 10$$

$$100 = -\frac{1}{2}D_1$$

$$D_1 = -200$$

$$100 = \frac{1}{4}D_2$$

$$D_2 = 400$$

$$-200 \leq D_1 \leq 10$$

$$D_2 = -20$$

$$-20 \leq D_2 \leq 400$$

$$D_3 = +20$$

$$-20 \leq D_3 \leq \infty$$

$$D_1 = 10$$

$$D_2 = -20$$

$$D_3 = -10$$

$$D_1 =$$

$$D_2 =$$

$$D_3 =$$

$$x_2 = 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2 = 100 + \frac{1}{2}(10) - \frac{1}{4}(-20) = 110$$

$$x_3 = 230 + \frac{1}{2}D_2 = 230 + \frac{1}{2}(-20) = 220$$

$$x_6 = 20 - 2D_1 + D_2 + D_3 = 20 - 2(10) - 20 - 10 = -30$$

$$Z = 1350 + D_1 + 2D_2 = 1350 + 10 - 40 = 1320$$

$$x_6 = -30 \quad \text{why?}$$

Because D_1, D_2, D_3 ranges are valid individually.
 Not valid when ranges are implemented simultaneously.

$$x_2 = 105 \quad D_1 = 30 \quad \text{not in range}$$

$$x_3 = 230 \quad D_2 = -12$$

$$Z = 100 + \frac{1}{2}(30) - \frac{1}{4}(-12)$$

$$x_2 = 118$$

$$x_3 = 230 + \frac{1}{2}(-12)$$

$$x_3 = 224$$

$$x_6 = 20 - 2(30) + (-12) + 100$$

$$x_6 = -40 - 12 + 100 = 48$$

$$Z = 1350 + 30 + 2(-12) = 1356$$

Case 2 → Change in Objective Function Coefficient.

$$Z = (3 + d_1)x_1 + (2 + d_2)x_2 + (5 + d_3)x_3$$

$$\text{Reduced cost for } x_1 = 4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1 \geq 0$$

$$\text{Reduced cost for } x_2 = 1 + \frac{1}{2}d_2 \geq 0$$

$$\text{Reduced cost for } x_3 = 2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 \geq 0$$

$$\text{Reduced cost for RHS value} = 1350 + 100d_2 + 230d_3$$

$$-2 \leq d_1 \leq 4 \quad 4 \geq \frac{1}{4}d_2 \quad 2 \geq \frac{1}{4}d_2$$

$$d_2 = 16 \quad -2 \leq d_2 \leq 8 \quad 4 \geq -\frac{3}{2}d_3$$

$$d_2 = -2 \quad d_3 = -8 \quad -2.67$$